pressures of 30-40 $\cdot 10^5$ N/m^2 in tank storage will be above normal, so that heating of the tube system outside the reactor may be required.

BF_3 has critical parameters -12.26°C and $49 \cdot 10^5$ N/m^2; it is stable with respect to temperature, at least up to 650° C, and does not react with steel when completely dry. Neutron absorption by boron is accompanied by production of lithium, which then reacts with fluorine forming LiF. To prevent corrosion of the tubes, an additive (e. g., ethylene) that will take up free fluorine may be introduced.

He^3 is stable at high temperatures and in all other respects is the most favorable gaseous absorber. Its production presents no difficulty: apart from a series of nuclear reactions, samples of helium strongly enriched in He^3 may be obtained from natural helium by gaseous diffusion and also by a method based on the superfluidity of helium. Neutron absorption in He^3 yields $_1H^3$ and liberates about 0.75 MeV, which corresponds to 20.2 billion joules per kg He^3.

The properties of gadolinium and samarium are given in [6] and elsewhere, and a description of the mechanics of aerosols in [5], etc.

Approximate calculations for some special cases show that when a control rod is replaced by a gas, the diameter of the gas channel must be commensurate with the diameter of the control rod channel (for equal lengths).

REFERENCES

1. W. Hübschmann, Atomkernenergie, no. 6, 226, 1959.
2. G. V. Samsonov, L. Ya. Markovskii, A. F. Zhigach, and M. G. Valyashko, Boron, Its Compounds and Alloys [in Russian], Izd-vo AN USSR, Kiev, 1960.
3. H. Booth and D. Martin, Boron Trifluoride and Its Derivatives [Russian translation], IL, 1955.
4. V. Keezom, Helium [Russian translation], IL, 1949.
5. N. A. Fuchs, Mechanics of Aerosols [in Russian], Izd-vo AN SSSR, 1955.
6. N. Trifonov, Rare-Earth Elements [in Russian], Izd-vo AN SSSR, 1960.

9 March 1964

All-Union Correspondence Polytechnic Institute,
Moscow

EXACT NUMERICAL SOLUTIONS OF THE BOUNDARY LAYER EQUATIONS FOR PSEUDO-PLASTIC FLUIDS

B. M. Berkovskii

Inzhenerno-Fizicheskii Zhurnal, Vol. 8, No. 4, pp. 528-530, 1965

The equation of the three-dimensional nonstationary boundary layer for fluids with rheology governed by a power law was derived in [1-3]. We shall consider certain exact solutions of self-similar problems of the boundary layer equations of pseudo-plastic fluids.

Flat permeable plate. We shall seek a solution of the two-dimensional stationary problem in the form

$$u = U_\infty \frac{dF}{d\eta} = U_\infty F', \quad \eta = y\left[\frac{U_\infty^{2-n}\rho}{n(n+1)kx}\right]^{\frac{1}{n+1}},$$
$$v = \frac{1}{1+n} x^{-\frac{n}{1+n}}\left[n(1+n)\,U_\infty^{2n-1}\frac{k}{\rho}\right]^{\frac{1}{1+n}}(\eta F' - F). \tag{1}$$

Then the equations of motion and the boundary conditions are

$$|F''|^{n-1}F''' + FF'' = 0,$$
$$F(0) = N, \quad F'(0) = 0, \quad \lim_{\eta\to\infty} F' = 1. \tag{2}$$

Numerical integration of system (2) enables one to determine the friction losses. The total friction drag

$$C_f = \frac{2b\int_0^L \tau_0\,dx}{{}^1/_2\,\rho U_\infty^2\,2bL} = 2(1+n)\,[n(1+n)]^{-\frac{n}{1+n}}\,|F''(0)|^n R^{-\frac{1}{1+n}}, \tag{3}$$

$$R = \rho\,\frac{U_\infty^{2-n}L^n}{k}.$$

The quantity F" (0) required for determining C_f is tabulated below for various n and several levels of injection and suction at the surface.

Steady two-dimensional flow near the stagnation point. A solution is sought in the form

$$u = U\frac{dF}{d\eta} = axF', \quad \eta = yx^{\frac{1-n}{1+n}}\left[\frac{2\rho a^{2-n}}{k(n+1)}\right]^{\frac{1}{1+n}},$$

$$v = -\frac{2na}{1+n}\left[\frac{a^{2n}k(1+n)}{2\rho}\right]^{\frac{1}{1+n}} x^{\frac{n-1}{1+n}}\left(F + \frac{1-n}{2n}\eta F'\right). \tag{4}$$

Then the equations of motion and the boundary conditions will be

$$|F''|^{n-1}F''' + FF'' + \frac{n+1}{2n}(1-F'^2) = 0,$$

$$F(0) = 0, \quad F'(0) = 0, \quad \lim_{\eta\to\infty} F' = 1. \tag{5}$$

Numerical solution of (5) enables one to find the local drag

$$c_f = \frac{\tau_0}{{}^1/_2\,\rho U^2} = 2\left(\frac{2}{1+n}\right)^{\frac{n}{1+n}} |F''(0)|^n R_x^{-\frac{1}{1+n}}, \quad R_x = \rho\,\frac{U^{2-n}x^n}{k}. \tag{6}$$

For flow of various pseudo-plastic fluids near the stagnation point the value of F" (0) for n = 1, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, is, respectively, 1.242, 1.256, 1.279, 1.316, 1.378, 1.484, 1.684, 2.132, 3.608.

Self-similar unsteady flow near the stagnation point. We shall seek a solution in the form

$$u = U\frac{dF}{d\eta} = a\frac{x}{t}F', \quad \eta = yx^{\frac{1-n}{1+n}}\,t^{\frac{n-2}{1+n}}\left[\frac{2\rho a^{2-n}}{k(n+1)}\right]^{\frac{1}{1+n}},$$

$$v = -\frac{2na}{1+n}\left[\frac{a^{2-n}k(1+n)}{2\rho}\right]^{\frac{1}{1+n}} x^{\frac{n-1}{1+n}}\,t^{\frac{1-2n}{1+n}}\left(F + \frac{1-n}{2n}\eta F'\right). \tag{7}$$

We write out the equations of motion and the boundary conditions

$$|F''|^{n-1}F''' + FF'' + \frac{1+n}{2n}(1-F'^2) +$$

$$+ \frac{1+n}{2na}\left(F'^2 - \frac{n-2}{1+n}\eta F'' - 1\right) = 0, \tag{8}$$

$$F(0) = 0, \quad F'(0) = 0, \quad \lim_{\eta\to\infty} F' = 1.$$

It is clear from (7) and (8) that at the initial time t = 0 the motion begins with infinite velocity. This self-similar problem may be treated as flow near the forward stagnation point of a blund impermeable body at a decreasing velocity of the external flow inversely proportional to time. The local drag is then given by

$$c_f = \frac{\tau_0}{{}^1/_2\,\rho U^2} = 2\left(\frac{2}{1+n}\right)^{\frac{n}{1+n}} |F''(0)|^n R_x^{-\frac{1}{1+n}} \quad R_x = \rho\,\frac{U^{2-n}x^n}{k},$$

where F" (0) is found from the solution of (8) and has the following values for various a and n: when n = 1, 0.9, 0.8, 0.7, 0.6, 0.5 and a = 100 F" (0) is, respectively, 1.230, 1.239, 1.253, 1.276, 1.313, 1.374, for the same values of n and a = 10, F" (0) = 1.204, 1.212, 1.226, 1.249, 1.285, 1.345, when a = 1, F" (0) = 0.9232, 0.9346, 0.9514, 0.9760, 1.013, 1.072.

Values of F"(0) for Flow Over a Flat Plate of Various Pseudo-Plastic Fluids at Several Levels of Injection and Suction

F (0)	Values of F" (0) when n =									
	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
−0.8	0.0175	0.0414	0.0594	0.0723	0.0806	0.0846	—	—	—	—
−0.5	0.1485	0.1533	0.1546	0.1527	0.1461	0.1376	—	—	—	—
0.0	0.4696	0.4339	0.3962	0.3567	0.3157	0.2735	0.2306	0.1883	0.1472	0.1088
+0.5	0.8579	0.7960	0.7270	0.6509	0.5687	0.4819	0.3933	0.3064	0.2257	0.1553
+1.0	1.284	1.214	1.129	1.028	0.9097	0.7764	0.6313	0.4831	—	—
+3.0	3.145	3.198	3.227	3.217	3.143	2.973	—	—	—	—

In all three cases the friction depends on k, n, and Re in a very complex way.

REFERENCES

1. Y. Tomita, Bull Jap., Soc. Mech. Engr., 4, 77, 1961.
2. J. N. Kapur, Journ. Phys. Soc. Japan, 17, 1303, 1962.
3. B. M. Berskovskii, DAN BSSR, no. 1, 1965.

Institute of Heat and Mass Transfer AS BSSR, Minsk